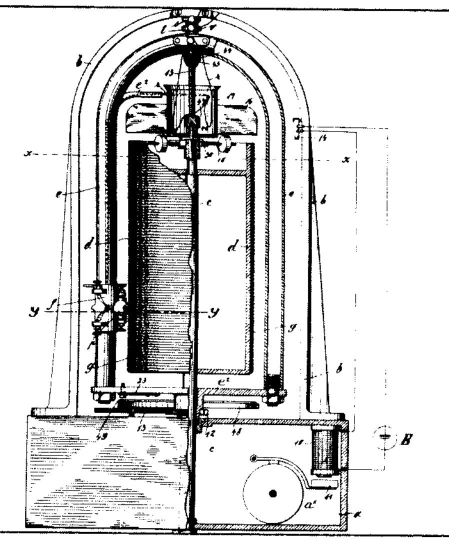
AUDIO TECHNICAL INFORMATION NUMBER 1



Maximum Signal-to-Noise Ratio of a Tape Recorder

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ABSTRACT

Using the Wiener auto-correlation theorem, the noise power spectrum of the pole strength in a thin lamina of an erased tape is shown to be approximately "white." The noise power spectrum of the reproduce head voltage is calculated for a thick tape and compared with the signal power. The wideband signal-to-noise ratio of a tape recorder equalized flat is deduced and expressed in very simple forms, which are inversely dependent upon the square of a bandwidth. Notably, in this special case the wideband result is independent of reproduce head-to-tape spacing. Numerical examples demonstrate that this simple theory yields results in excellent agreement with practice.

INTRODUCTION

The signal-to-noise ratio (SNR) of a tape recorder is, with the possible exception of the "drop-out" behavior, the most important factor governing its utility as an information storage system. The maximum possible SNR, which occurs when the principal noise source in the system is the tape itself, depends naturally not only upon the fundamental parameters of the tape but also upon the manner of its use. The discussions of SNR given previously,[1,2] though correct, seem to be needlessly complex. Further, the results are not in forms readily useable by the system designer. In the present paper the entire problem is reworked in a simple, direct manner using the Wiener auto-correlation theorem.

It is shown that the wideband SNR may be expressed in very simple forms which yield values in exceptionally close agreement with experiment. Several new relationships of practical significance are derived and discussed. Further, since all the important expressions are derived from first principles, it is believed that the work is not without pedagogic merit.

INITIAL CONSIDERATIONS

Whereas the signal in a tape recorder relates to the mean magnetization of the tape particles, the noise arises from the deviations from the mean of the magnetization. In an erased tape the major source of these deviations is the randomness of the particle magnetization directions. We shall assume that only two directions exist, positive and negative, which are occupied at random.

As the tape becomes magnetized and the directional randomness decreases, one might expect the noise to decrease. In fact, it increases somewhat, probably due to non-uniform particle packing effects. A noise which depends upon the signal (modulation noise) is neither stationary nor additive. However, since in the best tapes the noise increase is slight ($\approx$ 3-4 dB), we shall assume here that the noise is stationary and additive at all signal levels.

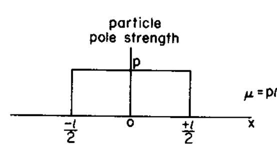

FIGURE 1. Particle pole strength.

TAPE MAGNETIZATION STATISTICS

We seek first the auto-correlation function (ACF), taken in the direction of head-to-tape motion (x), of the pole strength* in a lamina, of width ω and thickness δy, of an erased, oriented, particulate, tape. Suppose the single domain particles be identical, have dipole moment $\mu = pl$ (see Fig. 1) and be at a density n. The pole strength of the lamina, at longitudinal position x, is,

$$P(x) = \sum_i b_i \, p_i(x)$$

$$\text{where } b_i = \pm 1 \text{ at random} \qquad (1)$$

The ACF is, by definition for stationary random processes,[3]

$$\text{ACF}(x') = \lim_{X \to \infty} \frac{1}{X} \int_{-X/2}^{+X/2} P(x)\, P(x-x')\, dx \qquad (2)$$

$$= \lim_{X \to \infty} \frac{1}{X} \int_{-X/2}^{X/2} \sum_i b_i \, p_i(x) \sum_j b_j \, p_j(x-x')\, dx \qquad (3)$$

$$= n\, \omega\, dy \int_{-\infty}^{+\infty} p(x)\, p(x-x')\, dx$$

$$\text{since } \overline{b_i \, b_j} = 1 \text{ if } i = j$$
$$= 0 \text{ if } i \neq j$$

*The pole strength is defined by $P(x) = \int_A M(x)\, dA$, where M(x) is the magnetization and A is the cross sectional area.

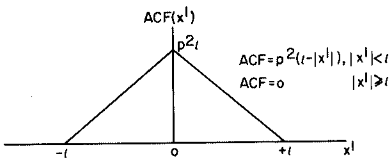

FIGURE 2. Auto-correlation function of particle pole strength.

Since, on the average, the particles only correlate with themselves, the lamina pole strength ACF is simply the sum of the individual particle pole strength ACF's, each of which is equal to $p^2 (l - |x'|)$ (see Fig. 2). According to the Wiener theorem the noise power spectrum is given by the Fourier cosine transform of the ACF.[4] Thus the noise power spectrum of the lamina pole strength is:

$$\Theta(k) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} n\,\omega\,dy\,p^2\,(l-|x'|)\cos kx'\,dx' \tag{5}$$

$$= \frac{\mu^2\,n\,\omega\,dy}{2\pi}\left[\frac{\sin\frac{kl}{2}}{\frac{kl}{2}}\right]^2 \tag{6}$$

This function is plotted in Fig. 3. Note that the important result that, for wavelengths (λ) substantially larger than the individual particle length, the lamina pole strength noise power spectrum is flat. This "white" spectrum approximation is assumed hereafter.

OUTPUT NOISE POWER SPECTRA (NPS)

Having defined the statistics, we proceed to compute the reproduce head voltage NPS. Providing gap losses may be neglected, the reproduce head exhibits a linear voltage transfer function $4\pi V |k| e^{-|k|y}$. That this is true may be seen immediately since

$$\int_a^{a+d} 4\pi V |k|\, e^{-|k|y}\, dy = 4\pi V (1-e^{-|k|d})\, e^{-|k|a} \tag{7}$$

which is the familiar Wallace output voltage spectrum.[5] To compute the output voltage NPS, we multiply the lamina pole strength noise power spectrum by the reproduce head power transfer function and integrate through

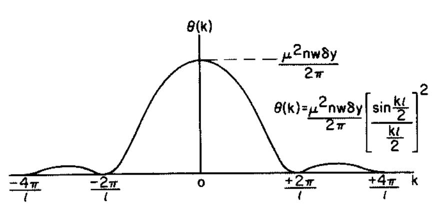

FIGURE 3. Noise power spectrum of lamina pole strength.

the tape thickness. This operation is, in physical terms, allowing for the fact that the reproduce head only senses a wavelength dependent, limited volume of tape adjacent to the gap.

Thus,

$$E_N^2(k) = \int_a^{a+d} \frac{\mu^2\,n\,\omega\,dy}{2\pi}\left[4\pi V |k|\, e^{-|k|y}\right]^2 \tag{8}$$

$$= 4\pi\,\mu^2\,n\,\omega\,V^2\,|k|\,(1-e^{-2|k|d})\,e^{-2|k|a} \tag{9}$$

a result obtained by both Daniel[1] and Stein.[2] A similar development using the transfer function for a non-differentiating head ($4\pi\,e^{-|k|y}$) leads to the output flux NPS.

$$\Phi_N^2(k) = \frac{E_N^2(k)}{V^2 k^2} = \frac{4\pi\,\mu^2\,n\,\omega\,(1-e^{-2|k|d})\,e^{-2|k|a}}{|k|} \tag{10}$$

Should expressions (9) and (10) be integrated over an infinite bandwidth despite the comments following equation (6), and the onset of reproduce gap losses, the results are:

$$\int_{-\infty}^{\infty} E_N^2(k)\,dk = 4\pi\,\mu^2\,n\,\omega\,V^2\left\{\frac{d\,(a+d/2)}{a^2\,(a+d)^2}\right\} \tag{11}$$

and

$$\int_{-\infty}^{\infty} \Phi_N^2(k)\,dk = 8\pi\,\mu^2\,n\,\omega\,\log_e\left(\frac{a+d}{a}\right) \tag{12}$$

as given by Mee.[6] They represent merely upper bounds to the total noise power. It will be evident that should exact results be needed they could be computed with little difficulty.

OUTPUT SIGNAL POWER SPECTRUM (SPS)

Suppose that, perhaps because of the need to minimize distortion, the sinusoidal signal

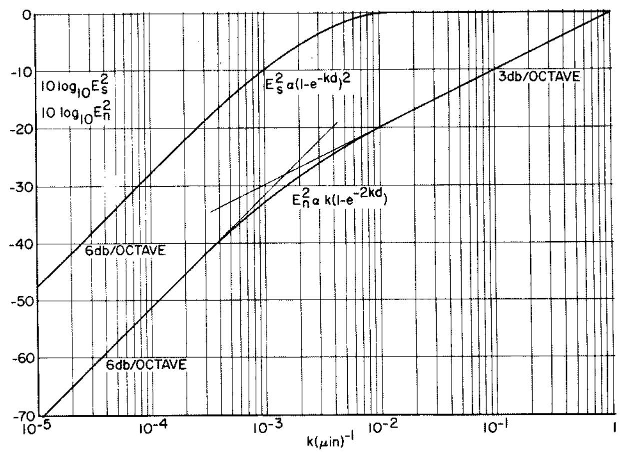

FIGURE 4. Relative signal and noise power spectra versus wavenumber (k) for a 400 μinch coating. No head-to-tape spacing effect is shown since it would change both curves equally.

magnetization recorded on the tape is only at a fraction f of the maximum amplitude possible. Further suppose, perhaps because of the need to minimize short wavelength record process losses, the tape is only recorded upon to a limited depth $d' \leq d$. Apparently by inspection of equation (7) the output signal power spectrum is:

$$E_S^2(k) = \tfrac{1}{2}\left[4\pi\,\mu\,n\,\omega\,f\,V\,(1-e^{-|k|d'})\,e^{-|k|a}\right]^2 \tag{13}$$

It will be noted the head-to-tape spacing dependence of both the SPS and NPS is identical. This occurs because the same physical laws govern both signal and noise of the same frequency. The two spectra are shown in Fig. 4.

The measured signal spectrum matches the calculated curve very closely. The measured noise spectrum[7] deviates appreciably at long wavelengths from that expected. In particular, the measured noise spectrum has a lower slope than expected. This is probably because the measurements unavoidably include "surface" noise (attributable to tape roughness and consequent head-to-tape spacing variations) the magnitude of which increases with decreasing frequency. However, the differences are small when the highest quality tape is used and in any case such low frequency differences have little effect upon the wideband SNR.

NARROW-BAND SNR

The narrow-band SNR for a "slot" of width Δk is:

$$(SNR)_{narrow} = \frac{2\pi\,n\,\omega\,f^2\,(1-e^{-|k|d'})^2}{|k|\,(1-e^{-2|k|d})\,\Delta k} \tag{14}$$

It is, of course, independent of head-to-tape spacing. The adverse effects of nonsaturation and partial penetration recording are evident; both reduce the SNR because, whilst only a limited number of particles contribute to the signal, all still contribute to the noise.

WIDEBAND SNR

Since the signal and noise power spectra are not identical, the wideband SNR depends upon the reproduce system equalization. Generally, wideband SNR's will also depend upon the head-to-tape spacing. A simple case, of particular interest because of its widespread use, occurs when the output signal is equalized "flat." To achieve this, the power transfer function of all parts of the reproduce system after the head must be the reciprocal of the signal power spectrum given by equation (13).

Note that this particular equalization makes the equalized noise power spectrum independent of head-to-tape spacing. An important consequence is that, in this special case, the wideband SNR is independent of reproduce head-to-tape spacing. "Out of contact" playback need not entail a loss in SNR providing other noises in the system are kept below the (attenuated) tape noise. The onus is on the system designer.

The wideband SNR for such systems is customarily defined to be the equalized signal power divided by the integrated noise power in the system bandwidth. That is,

$$(SNR)_{wide} = \left[\int_{|k|_{min}}^{|k|_{max}} \frac{(1-e^{-2|k|d})\,|k|\,d\,|k|}{2\pi\,n\,\omega\,f^2\,(1-e^{-|k|d'})^2} \right]^{-1} \quad (15)$$

Before evaluating this expression, two simplifications may be mentioned. First, if we consider only full coating depth recording ($d' = d$), then, dropping the modulus signs,

$$(SNR)_{wide} = 2\pi\,n\,\omega\,f^2 \left[\int_{k_{min}}^{k_{max}} k \coth\frac{kd}{2}\,d\,k \right]^{-1} \quad (16)$$

The signal and noise spectra for this case are shown in Fig. 5. Second, for a system in which the wavelengths over a substantial fraction of the bandwidth are comparable to or smaller than the tape coating thickness, so that $kd \gg 1$ and $\coth kd/2 \approx 1$, then

$$(SNR)_{wide} \approx 4\pi\,n\,\omega\,f^2 \left[k^2_{max} - k^2_{min} \right]^{-1} \quad (17)$$

This form may be compared with that resulting from the common, but erroneous, assumption that the tape noise is "white" in which case SNR α $(k_{max} - k_{min})^{-1}$. In a system equalized flat, the NPS rises at approximately 3 dB/octave, and consequently doubling the bandwidth actually entails a loss in SNR of about 6 dB rather than 3 dB.

It will be shown below that the extremely simple form of equation (17) does indeed closely approximate measured SNR's. It should be noted that the tape speed (V), the head-to-tape spacing (a), the coating thickness (d) and the dipole moment (μ) do not appear; they do not have an important effect upon the maximum SNR.

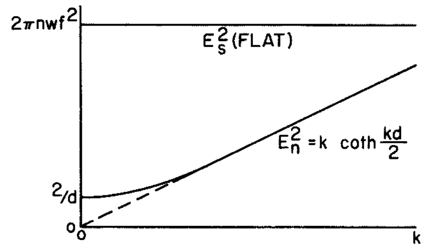

FIGURE 5. Signal and noise power spectra versus wavenumber (k) in a system equalized flat. The depth of recording is equal to the coating thickness. Both spectra have been multiplied by a factor of $2\pi n \omega f^2$.

The best tapes obviously yield the highest product of nf^2. Magnetostatic interparticle interactions, which are rather poorly understood, control the distortion limit (f) and consequently n and f are not independent variables. No simple theory giving the functional dependence of f on n can presently be given.

NUMERICAL EXAMPLES

We consider first the case of a 400-Hz to 1.5-MHz 120-ips, 50-mil trackwidth, wideband analog recorder equalized flat. The record gap length (150 μin) used is known to be noncritical, since the adjustments of the input currents largely compensate for differing gap lengths. Both the reproduce gap length (25 μin) and the average γ-Fe_2O_3 particle length (about 20 μin) are much smaller than the minimum wavelengths occurring (80 μin). A-c bias is used at a level which yields the maximum short wavelength output. If a head-to-tape spacing of 20 μin is assumed, the unequalized signal spectrum matches that expected for a partial penetration depth of about 75 to 100 μin. The signal input level is adjusted so that no more than 1% third harmonic distortion exists at long wavelengths. Under similar conditions, the RMS remanent flux in audio tapes has been found to be about 200 nano-weber per meter of track width which is equivalent to a peak magnetization of about 250 gauss.[8] Since the maximum remanence of γ-Fe_2O_3 analog tapes is about 1250 gauss, the distortion limit (f) is taken to be 0.2. The tape (Ampex 771) of coating thickness 400 μin, contains acicular γ-Fe_2O_3 particles of dimensions $20 \times 4 \times 4$ μin (i.e., 5000 x 1000 x 1000 Å) which are packed at one-third by volume. The number of particles per cubic microinch (n) is therefore about 10^{-3}.

The exact SNR given in equation (15) may be written:

$$(SNR)_{wide} \approx 2\pi\,n\,\omega\,f^2 \left[\frac{1}{d^2} \int_0^u \frac{s(1-e^{-2s})\,d\,s}{(1-e^{-\alpha s})^2} \right]^{-1} \quad (18)$$

where $u = k_{max}\,d$, and $\alpha = d'/d$

In the present case, substituting numbers,

$$(SNR)_{wide} \approx 2.10^6 \left[\int_0^u \frac{s(1-e^{-2s})\,d\,s}{(1-e^{-\alpha s})^2} \right]^{-1} \quad (19)$$

$$\text{with } u = \frac{2\pi\,d}{\lambda_{min}} \approx 30, \text{ and } \alpha = 0.4$$

The integral has been evaluated numerically and the results are tabulated in Table 1. Consequently, equation (19) may be written

$$10 \log_{10} (SNR)_{wide} = 10 \log_{10} (2.10^6) - 26.6 = 63 - 26.6 = 36.4 \text{ dB}$$

The simple approximate form given in equation (17) yields

$$10 \log_{10} (SNR)_{wide} = 10 \log_{10} (4\pi\,n\omega f^2\,k_{max}^{-2}) = 10 \log_{10} (4000) = 36 \text{ dB}$$

Table I $\quad 10 \log_{10} \int_0^u \frac{s(1-e^{-2s})\,d\,s}{(1-e^{-\alpha s})^2}$

for some values of u and α

u	α 0.2	0.4	0.6	0.8	1.0
5	20.0	15.5	13.6	12.5	12.0
10	22.1	18.9	17.9	17.5	17.3
20	25.2	23.6	23.3	23.1	23.1
40	29.7	29.2	29.1	29.1	29.0
80	35.2	35.1	35.1	35.1	35.1
160	41.1	41.1	41.1	41.1	41.1

which is less than 1 dB different from the exact result. Experimentally, if due care is taken to minimize other noises (mainly those due to reproduce head eddy currents) and to maintain the head efficiency at the upper frequencies, wideband RMS signal-to-RMS noise ratios of 34 to 35 dB have been measured in excellent agreement with the above theory.

As a second example we consider briefly a 40-Hz to 15 KHz, 7.5-ips, 80-mil trackwidth, professional audio recorder. Such machines use both variable pre-equalization (of the record current) and fixed post-equalization, whereas the above theory considers only variable post-equalization.

It might seem, therefore, that the theory is not directly applicable. However, it turns out that direct application of equations (15) and (17) in fact does yield "good" numbers when the tape speed is greater than or equal to 7.5 ips. This coincidence is related to the following considerations: the better quality tapes need little pre-equalization and thus produce an output spectrum close to that given by equation (7); the poorer tapes have considerable pre-equalization applied but again they yield the same output spectrum; and the fact that there is not much difference between the noise spectra of the different tapes.

To proceed with the calculation then we note that whereas the distortion limit is the same as in the previous example, now the depth of recording is equal to the standard γ-Fe_2O_3 coating thickness (400 μin). In this case (u = 5, α = 1.0) equations (15) and (17) yield 10 log $(SNR)_{wide}$ values of 54 and 55 dB respectively, which values compare favorably with the 56 to 57 dB usually measured on such "half-track" audio machines.

The uncertain factors in these calculations are, of course, the partial penetration depth (d') and the distortion factor (f). Whereas the calculated $(SNR)_{wide}$ is not sensitively dependent upon the exact value of the penetration depth, it does depend critically upon the distortion factor used. The value adopted here (0.2) is believed to be quite accurate and typical of modern analog tapes. However, even if the distortion factor is regarded simply as an adjustable parameter, the valuable fact remains that the theory, with f = 0.2, yields results in such excellent agreement with practice.

The above theory does not consider the effects of magnetostatic interactions which, particularly in non-uniformly packed tapes, will give rise to modulation noise. The excellent agreements found using the above simple theory indicate, however, that, at least in the case of distortion limited recorders where, perforce, the signal level and tape magnetization is low, the effect of modulation noise upon $(SNR)_{wide}$ is small.

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Nomenclature:

A	cross sectional area of tape (normal to head-tape motion)
a	head-to-tape spacing
b	dimensionless factor equal to ±1
d	tape coating thickness
d'	depth of recording (d' ≤ d)
E_N	reproduce head noise voltage (k domain)
E_S	reproduce head signal voltage (k domain)
f	ratio of signal to maximum possible signal
k	wavenumber ($2\pi/\lambda$)
k_{min}	minimum wavenumber
k_{max}	maximum wavenumber
l	magnetic particle length
M	tape lamina longitudinal magnetization (x domain)
n	number of particles per unit volume
p	magnetic particle pole strength (x domain)
P	tape lamina pole strength (x domain)
S	dimensionless factor (kd)
u	dimensionless factor (k_{max} d)
V	head-to-tape relative velocity
ω	track width
x	tape longitudinal coordinate
x'	offset tape coordinate
y	tape normal coordinate
α	dimensionless factor (d' /d)
λ	wavelength
μ	magnetic particle dipole moment (pl)
Θ	lamina pole strength noise power (k domain)

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The drawing pictured on the front page is of the first magnetic recorder—the Telegraphone. The inventor, Valdemar Poulsen, received a U.S. patent approval for his "device for effecting the storing up of speech or signals by magnetically influencing magnetizable bodies" on November 13, 1900.

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